BELLCOMM, INC.

ABSTRACT

In an atmosphere of only exponential density variation with height, control moment gyros can maintain a symmetrical spacecraft in the inertial orientation without bias momentum accumulation and the attendant expenditure of RCS propellant. At altitudes of interest for the AAP-3/AAP-4 mission, however, the atmospheric density is "bulged" towards the sun and a small bias momentum will be accumulated in the CMG's with each orbit. This momentum accumulation will be in addition to that resulting from the gravitational torques on the non-symmetrical spacecraft.

An analysis was made of the effects of a non-uniform atmospheric density on CMG bias momentum accumulation and RCS propellant expenditure; the effect of different average atmospheric densities was also considered.

For an average atmospheric density of 4.5×10^{-15} slugs/ft^3 the bias moment accumulation resulting from a non-uniform atmosphere was calculated to be 25.6 ft-lb-min/day and the attendant RCS propellant expenditure required for CMG unloading is calculated to be 0.19 lb/day at a specific impulse of 275 sec.

SUBJECT: AAP-3/AAP-4 Bias Momentum
Accumulation and RCS Pro-
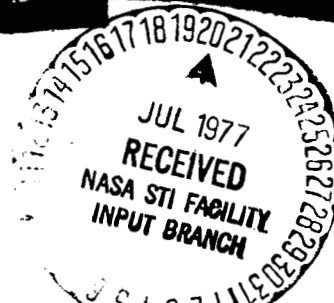pellant Expenditure Result-
ing from a Non-Uniform
Atmospheric Density
Case 600-3

DATE: July 20, 1967

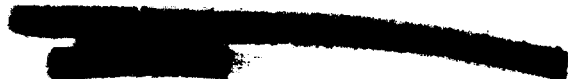FROM: J. W. Powers

MEMORANDUM FOR FILE

1.0 INTRODUCTION

In order to determine the control system torque required
to maintain the attitude of an orbiting spacecraft, the effects of
both induced and natural perturbations must be considered. Induced
perturbations can include the following factors:

- Attitude control reaction jets

- Control moment gyros (CMG's)

- Momentum transfer within the spacecraft (astronaut
and mechanical motion)

- Spacecraft atmosphere leakage, APU exhaust, etc.

Natural perturbations can include the following factors:

- Aerodynamic forces

- Geometry and mass distribution of Earth (gravi-
tational effects)

- Effects of other solar system bodies

- Solar radiation pressure

- Meteoroids

- Electromagnetic forces

In low Earth orbits, the most significant natural pertur-
bations result from aerodynamic and Earth non sphericity effects.

If the spacecraft is non-symmetrical, it will also be subject to a gravitational torque. This torque is further discussed in Section 4.0.

This memorandum will consider the effects of aerodynamic forces on an AAP-3/AAP-4 spacecraft in a low Earth circular orbit. The bias momentum accumulated and the reaction control system (RCS) propellant required to maintain the CMG equipped spacecraft in an inertial orientation in a non-uniform atmosphere excluding the gravitational effects will be determined. The bias momentum accumulation and RCS propellant consumed for different average atmospheric densities considering the atmospheric "bulge" will be determined.

2.0 AERODYNAMIC FORCE

The aerodynamic force (F_A) acting on a body is proportional to the product of the dynamic pressure $(\frac{\rho}{2} V^2)$ and the spacecraft projected cross sectional area (A). The aerodynamic moment (M_A) is the product of the aerodynamic force $(\frac{\rho}{2} V^2 C_D A)$ and the normal distance (d) from the resultant force vector to the spacecraft center of mass.

$$M_A = \frac{\rho}{2} V^2 C_D A \, d, \tag{1}$$

where ρ is atmospheric mass density, V is spacecraft velocity, and C_D is drag coefficient.

The total distributed pressure acting on the spacecraft may be conveniently considered as a resultant aerodynamic force acting at the spacecraft center of pressure (CP). For a body surface exposed to a uniform pressure field, the CP occurs at the centroid of the projected area.

3.0 ATMOSPHERIC DENSITY

Satellite orbital observations and detailed calculations have confirmed the existance of large diurnal atmospheric density variations. Figure I shown the density variation at 500 KM;[1][2] this curve shows a bulge occurring in the general direction of the Sun and lagging the Sun by approximately 30° easterly. The atmospheric density curve is also subject to variation with changes in the solar flux.[2]

The exact nature of the density variation with changing latitude is not known. If the shape of the density curve at a given altitude in the vicinity of the equator is as shown, either the "bulge" must flatten out as the latitude approaches 90° north and south, or density discontinuities in the vicinity of the poles must exist. A reasonable assumption is that the bulge occurs in both the longitude and latitude directions.

For the purposes of this discussion it will be assumed that the atmosphere density variation is as shown in Figure 1 for all latitudes that the AAP-3/AAP-4 spacecraft orbit transverses. Consideration of different average densities at the orbital height is accomplished by assuming that the shape of the curve is similar with changing average atmosphic density.

4.0   SPACECRAFT ORIENTATION

The AAP-3/AAP-4 mission is presently planned to be flown in an inertial mode with the longitudinal S-IVB axis (minimum inertia) in the orbital plane. The ATM axis is normal to the spacecraft axis, and sun pointing is accomplished by rotations about both the longitudinal axis and an axis thru the CG which is normal to the orbital plane.

If the spacecraft is symmetrical, the products of inertia are zero and the two transverse principal moments of inertia are equal. With this spacecraft configuration the gravitational torques will be zero if the longitudinal (minimum inertia) axis is maintained inertially in the orbital plane.

Both the magnitude and direction of the aerodynamic force and torque are affected by the spacecraft orbital orientation since different cross sectional areas and moment arms can be presented relative to the pressure field.

If the spacecraft maintains an inertial orientation the aerodynamic force experiences a cyclic change from maximum to minimum every 90° of orbital travel as a consequence of the change of vehicle projected area in the direction of the velocity vector. The moment arm also experiences a cyclic change from a maximum to zero. If the CP is located on the axis of minimum moment of inertia, (symmetrical spacecraft) the moment arm changes from a maximum to zero during the same cycle the aerodynamic force is changing from a maximum to minimum. The torque vector also reverses direction every 180° of orbital travel after becoming zero.

If a symmetrical vehicle is equipped with CMG's of sufficient momentum storage capacity, and the atmospheric density is constant, no RCS propellant will be required to maintain an inertial hold. With this orientation, the CMG's will experience a cyclic loading and unloading during each orbital period. If the atmospheric density varies during the orbital travel, however, a CMG momentum accumulation will occur; Unloading the CMG's will require periodic RCS thruster firing with an attendant propellant weight penalty.

The actual vehicle will of course not be symmetrical (finite products of inertia and non equal transverse principal inertias) by virtue of its complex geometry. With the actual configuration, a bias momentum accumulation which increases with time in orbit will occur. This effect will not be considered since the purpose of the memorandum is to isolate the effects of the non-uniform atmosphere on the spacecraft.

5.0 ANALYSIS

If $\rho = \rho(\theta)$, $A = A_o \cos \theta$ and $d = d_o \cos \theta$ where θ is the angle measured from some reference position when the projected area and torque arm are maximums, the change in direction and magnitude of moment is obtained from equation (2):

$$M_A = \frac{\rho(\theta)}{2} V^2 C_D A_o d_o |\cos \theta| \cos \theta \qquad (2)$$

if the moment M_o is known at θ_o, when the projected area and torque arm are maximums, equation (2) may be written:

$$\frac{M_A}{M_o} = \left[\frac{\rho(\theta)}{\rho_o} \right] |\cos \theta| \cos \theta$$

This function is shown in Figure II where $\rho = \rho(\theta)$ is taken from Figure I. The average moment ratio, $(\frac{M_A}{M_o})$ acting over the

orbital period can be determined from the algebraic sum of the positive and negative areas of Figure II. If this area is written:

$$\Delta \left[\int \frac{M_A}{M_o} \, d\theta \right]$$

the average moment ratio may be written:

$$\frac{M_A}{M_o} = \frac{1}{2\pi} \quad \Delta \quad \left[\int \frac{M_A}{M_o} \, d\theta \right] = 0.345, \tag{4}$$

where the numerical value is calculated from Figure II.

A detailed calculation shows a AAP-3/AAP-4 moment of approximately 0.1 Ft-Lb with an atmospheric density of 8.72×10^{-15} slug/ft^3. Since aerodynamic moment is proportional to density, equation (4) may be written:

$$M_A = \left(\frac{0.1 \, \rho_o}{8.72 \times 10^{-15}} \right) 0.345 = 0.0356 \left(\frac{\rho_o}{9 \times 10^{-15}} \right), \text{Ft-Lb}, \tag{5}$$

Equation (5) is converted to momentum accumulation (ΔH) by multiplying by time. (i.e. the number of minutes per day)

$$\Delta H = 51.2 \left(\frac{\rho}{9 \times 10^{-15}} \right), \quad \frac{\text{Ft-Lb-Min}}{\text{Day}} \tag{6}$$

To determine the weight of RCS propellant consumed per unit time, the burn time (t_b) of the thruster per unit time in orbit must be evaluated.

$$(F\ell)\ t_b = M_A \tag{7}$$

Where F is RCS thrust, (100 Lb), and ℓ is the distance from thruster center line to the center of mass, (approximately 29.2 ft). The RCS thrust may be written in terms of specific impulse (I_{sp}) and propellant weight flow rate ($\dot{w}$):

$$F = I_{sp}\ \dot{w} \tag{8}$$

Combining (7) and (8) and solving for propellant weight (W) required per day

$$W = \dot{w}\ t_b = \frac{M_A}{I_{sp}\ \ell}\ 60 \times 60 \times 24, \tag{9}$$

Using minimum specific impulse (130 sec) and Equation (5)

$$W = 0.81 \left(\frac{\rho}{9 \times 10^{-15}}\right), \frac{Lb}{Day} \tag{10}$$

Equations (6) and (10) are shown in Figure III.

CONCLUSIONS

The bias momentum acculumation and RCS propellant required to maintain the AAP-3/AAP-4 spacecraft in an inertial orientation

considering only a non-uniform atmosphere are presented. For expected average densities of 2 to 6 x 10^{-15} slugs/ft^3 the bias momentum accumulation is 11 to 34 $\frac{\text{Ft-Lb-Min}}{\text{Day}}$ and the required RCS propellant is 0.18 to 0.54 Lb/Day. The RCS propellant consumption is conservative since it is based upon the minimum specific impulse bit. In practice, the CMG's would be unloaded by continuous firing of the appropriate thruster units until the required counter momentum is generated. The required propellant weight will be less than that indicated in equation (10) by the factor $(\frac{130}{I_{sp}})$, where I_{sp} is the specific impulse over the unloading time interval. RCS thruster burn time is shown to be 1.054 $(\frac{\rho}{9 \text{ x } 10^{-15}})$ seconds per day from equations (5) and (7). An atmospheric density of 4.5 x 10^{-15} slugs/ft^3 yields an approximate thruster burn time of 0.5 sec/day. This burn time yields a propellant specific impulse of about 275 sec. This higher specific impulse indicates a probable RCS propellant consumption of approximately one half that indicated by equation (10).

1022-JWP-mef

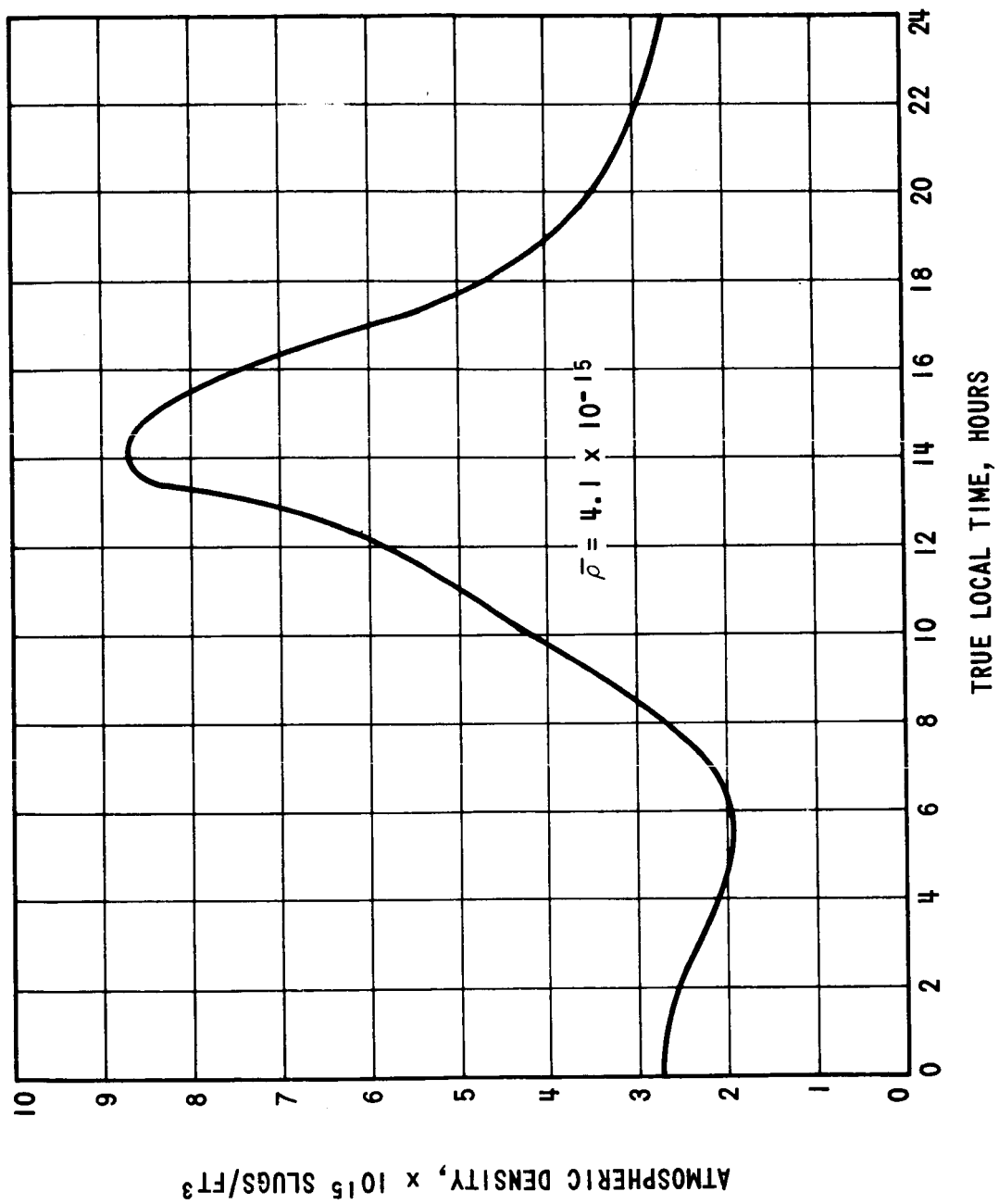J. W. Powers

Attachments
   Figures I through III
   References

ATMOSPHERIC DENSITY, × 10^15 SLUGS/FT³

TRUE LOCAL TIME, HOURS

$\bar{\rho} = 4.1 \times 10^{-15}$

FIGURE 1 - MASS DENSITY vs TRUE LOCAL TIME 500 KM ALTITUDE 20° > |LATITUDE|
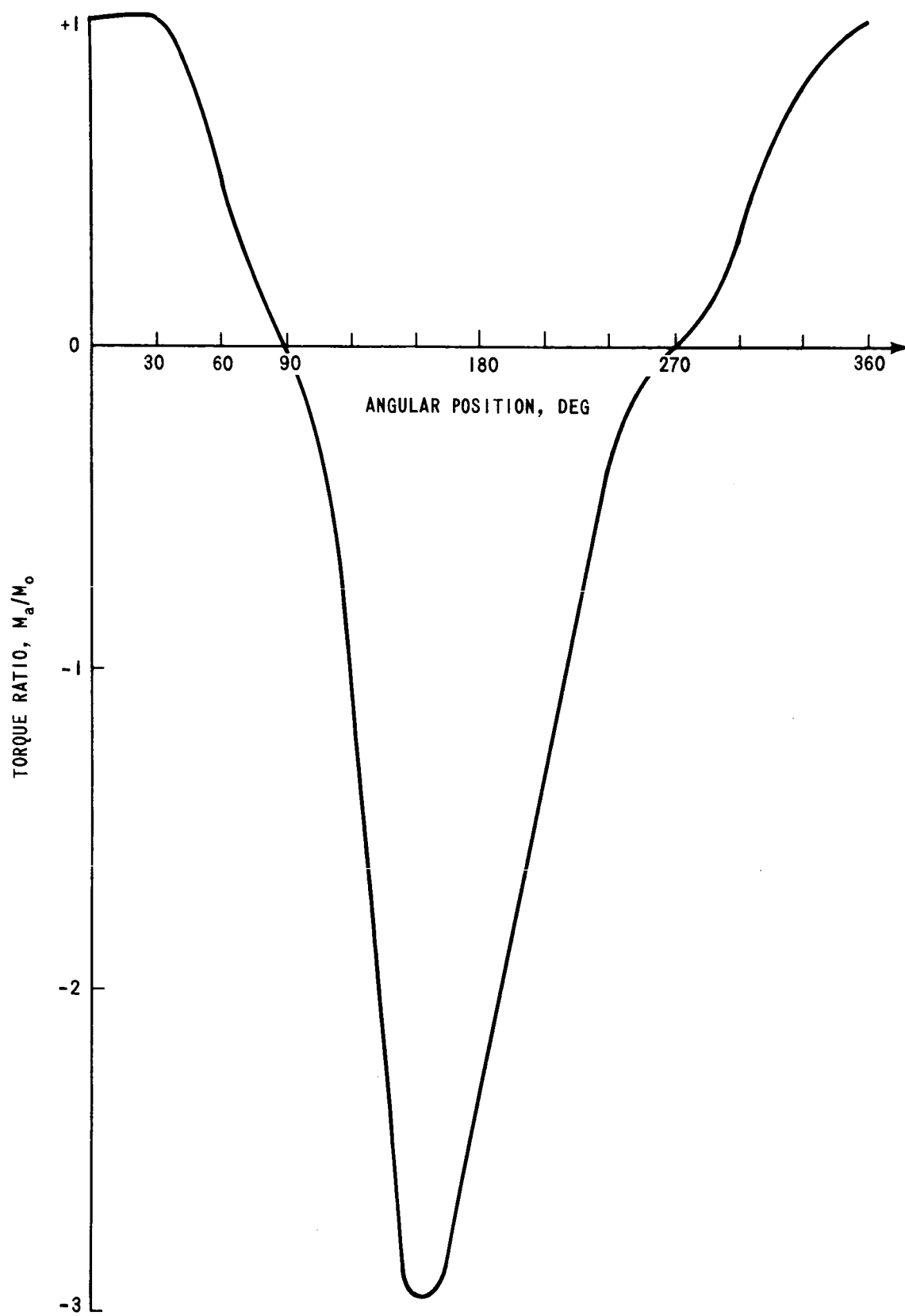20 CM SOLAR FLUX RADIATION S = 170 × 10^{-2} w/m² - CPS
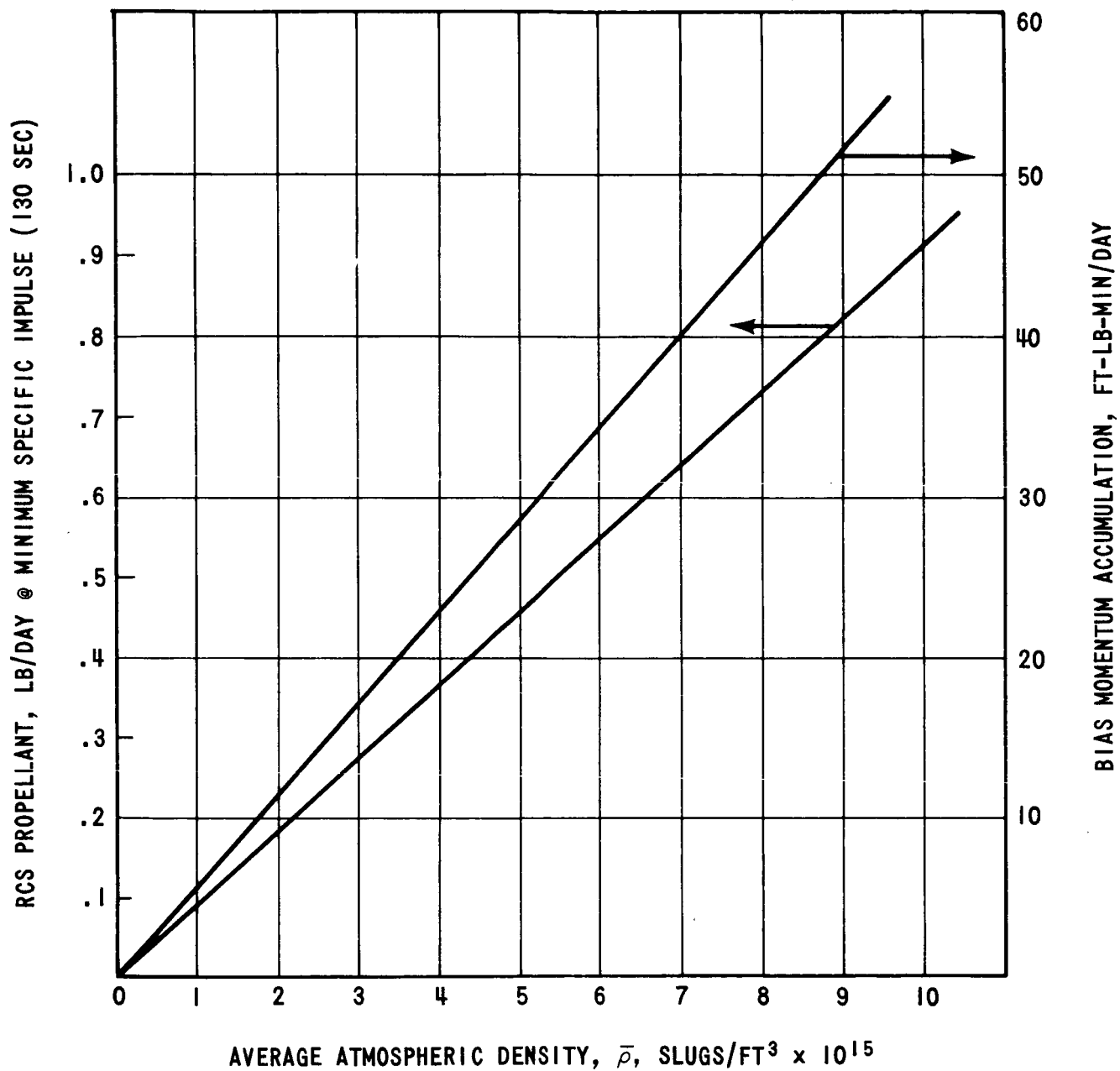
FIGURE II - TORQUE RATIO vs ORBITAL ANGULAR POSITION

FIGURE III - RCS PROPELLANT & BIAS MOMENTUM ACCUMULATION FOR AAP-ATM INERTIAL HOLD & DIURNAL ATMOSPHERIC VARIATION @ 500 KM FOR AVERAGE DENSITY, $\bar{\rho}$

BELLCOMM, INC.

REFERENCES

(1)   NASA SP-33, <u>Orbital Flight Handbook</u>, Figure 12 (II-75), 1963.

(2)   S. Glasston, <u>Source Book of the Space Sciences</u>, Figure 8.7.

BELLCOMM, INC.

Subject: AAP-3/AAP-4 Bias Momentum Accumulation   From: J. W. Powers
and RCS Propellant Expenditure Result-
ing from a Non-Uniform Atmospheric
Density

Distribution List

NASA Headquarters

Messrs.   H. Cohen/MLR
P. E. Culbertson/MLA
J. H. Disher/MLD
J. A. Edwards/MLO
L. K. Fero/MLV
J. P. Field, Jr./MLP
T. A. Keegan/MA-2
C. W. Mathews/ML
M. Savage/MLT
W. C. Schneider/MLM

Bellcomm

Messrs.   A. P. Boysen
D. R. Hagner
W. C. Hittinger
B. T. Howard
J. Z. Menard
I. D. Nehama
I. M. Ross
R. L. Wagner
Div. 101 Supervision
Div. 102 Supervision
All Members Depts. 1021, 1022, 1024
Department 1023
Central File
Library